MODULE-1

- SSB for U(1) Global gauge symmetry
- Higgs Mechanism for U(1) Abelian Gauge Symmetry
- Higgs Mechanism for Non-Abelian Gauge Theory

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Spontaneous Symmetry Breaking (SSB) of U(1) global gauge symmetry

- What is $U(1)$ global gauge symmetry?

$U(1) = e^{i\theta}$; where θ is independent of $(t, \vec{x})$

If $\mathcal{L}$, the Lagrangian density remains invariant under $U(1)$ global gauge group, we call it global $U(1)$ gauge invariant Lagrangian.

Here we shall consider the Lagrangian for the complex scalar field given by:

$$\mathcal{L} = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2$$

Under $U(1)$ global gauge transformation given by, $U(1) = e^{i\theta}$, the above Lagrangian remains invariant.

The "potential" : $V(\Phi, \Phi^\dagger) = \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$

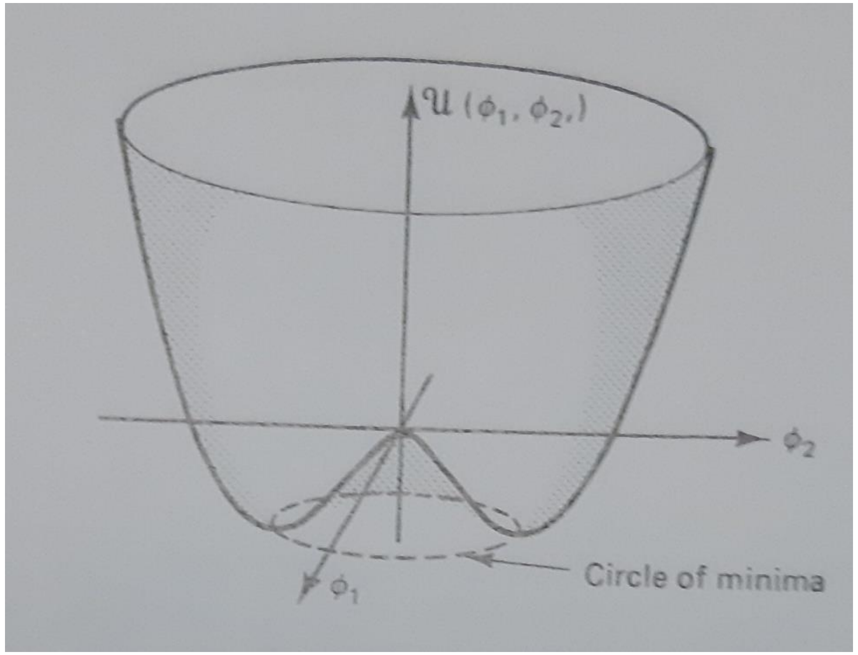
Problem: Assuming $\Phi = \frac{1}{\sqrt{2}} (\Phi_1 + i\Phi_2)$ and $\Phi^\dagger = \frac{1}{\sqrt{2}} (\Phi_1 - i\Phi_2)$ one can show that

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \Phi_1\right)(\partial^\mu \Phi_1) + \frac{1}{2}(\partial_\mu \Phi_2)(\partial^\mu \Phi_2) - \frac{\mu^2}{2}(\Phi_1^2 + \Phi_2^2) - \frac{\lambda}{4}(\Phi_1^2 + \Phi_2^2)^2$$

Let us define the magnitude of the vacuum expectation value (VEV) to be $|\langle 0|\Phi|0\rangle| = \frac{v}{\sqrt{2}}$; $v = \pm\sqrt{\frac{-\mu^2}{\lambda}}$.

*** Note that, here $\frac{v}{\sqrt{2}}$ is considered instead of v due to the normalization factors present in the definitions of Φ and $\Phi^\dagger$.

$\langle 0|\Phi|0\rangle = \frac{v}{\sqrt{2}} e^{i\zeta}$, here ζ is the parameter which can run continuously from 0 to 2π. All values of ζ are valid and if different values are plotted we obtain a circle. All the minima lie on the circumference of circle. The potential profile corresponding to the potential looks like :

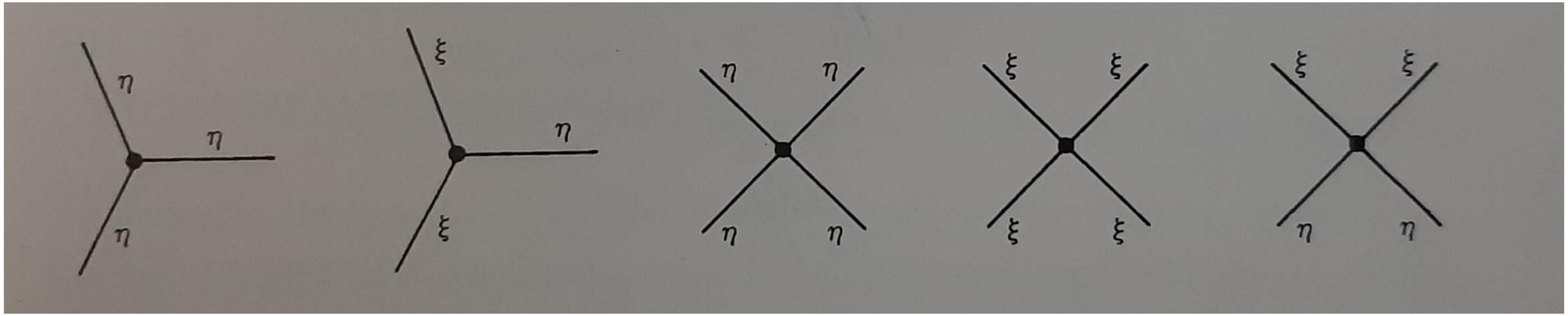


The symmetry will be spontaneously broken when one of the minima out of infinite minima is chosen by the system. All the minima will lie on the circle of minima indicating degenerate energy eigen states. Since, Φ is complex, we can write , $\Phi = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$ where, $\eta(x)$ and $\xi(x)$ are newly introduced fields.

Rewriting the Lagrangian in terms of these new field variables, we find :

$$\mathcal{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \mu^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\right] + \left[\mu\lambda(\eta^3 + \eta\xi^2) - \frac{\lambda}{4}(\eta^4 + \xi^4 + 2\eta^2\xi^2)\right] + \frac{\mu^4}{4\lambda}$$

- First term is free K. G. Lagrangian for the field η, which carries a mass $\mu\sqrt{2}$
- Second term is free Lagrangian for the field ξ, which is massless.
- Third term defines five couplings:



Observations:

- DOF matching: Before SSB, DOF was 2 (Φ_1 & Φ_2) and after SSB it became 2 (η & ξ)
- The new form of Lagrangian does not posses the symmetry of the older one. It is spontaneously broken by the selection of a particular vacuum state.
- One of the newly defined fields (ξ) becomes massless keeping the other one (η) massive.
- According to ***Goldstone's Theorem*** it is known that spontaneous breaking of a continuous global symmetry is always accompanied by the appearance of one or more massless scalar (spin-0) particles which are called ***Goldstone bosons***.

Take home message:

Although we started our journey hoping to use SSB to account for the mass of weak interaction gauge fields, but we landed up with a massless scalar boson which had not yet been detected in any high energy particle physics experiments. It is hard to believe that a massless scalar boson would miss all the detectors without depositing energy in any form of detectors. If neutral they should show up in the form of missing energy and momentum.

Let us apply the idea of SSB to the case of ***local*** gauge invariance . It woks fantastically here and resolves the issue of giving masses to the gauge fields.

HIGGS MECHANISM

(Anderson – Brout – Englert – Guralnik – Hagen – Higgs - Kibble)

(U(1)-abelian gauge theory)

Consider local $U(1)$ gauge invariance with $U(1) = e^{i\theta(x)}$ with θ being space-time dependent variable. Under this transformation the field Φ transforms like : $\Phi' = e^{i\theta(x)}\Phi$, the following Lagrangian remains invariant as long as θ is global variable.

$$\mathcal{L} = (\partial_\mu\Phi)^\dagger(\partial^\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$$

The story takes a remarkable twist if one considers local $U(1)$ gauge invariance .The prescription is same as the case of Spinor fields. In order to restore the local gauge invariance of the above Lagrangian one needs to introduce a massless gauge fields A_μ, and replacing the derivatives of the above equation by covariant derivatives :

$$\mathcal{D}_\mu = \partial_\mu + igA_\mu$$

The transformation of the gauge field is given by: $A'_\mu = A_\mu + \frac{1}{g}\partial_\mu\theta$

The generic form of Lagrangian for abelian gauge theory is given by:

$$\mathcal{L} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Where, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

This is the most attractive mechanism for explaining the origin of mass of elementary particles. Actually it gave the answer to the longstanding question : What is the origin of mass of elementary particles?

Higgs Mechanism for U(1) Abelian gauge group

Here we shall consider $U(1)$ local Abelian gauge transformation defined by $U(1) = e^{ig\theta(x)}$; where g is the $U(1)$ gauge coupling and θ is function of $(t, \vec{x})$.

Local gauge invariance introduces the gauge field which is responsible for mediating interactions. The gauge field is denoted by A_μ . According to the prescription, to keep the Lagrangian invariant under the local $U(1)$ gauge group one needs to replace ordinary derivatives (∂_μ) by covariant derivatives $(\mathcal{D}_\mu)$.

Therefore, we can write the $\mathcal{L}$, the Lagrangian density for complex scalar field, which remains invariant under $U(1)$ local gauge group as :

$$\mathcal{L} = (\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}^\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad \text{-----------------------------} \quad (1)$$

Where, $\mathcal{D}_\mu = \partial_\mu - igA_\mu$; $\Phi = \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2)$ and $\Phi^\dagger = \frac{1}{\sqrt{2}}(\Phi_1 - i\Phi_2)$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

The transformation of fields under $U(1)$ local gauge group are defined by :

$\Phi'(x) = e^{ig\theta(x)}\Phi(x)$ and and $A'_\mu(x) = A_\mu(x) + \frac{1}{g}\partial_\mu\theta$

- 2nd and 3rd terms in Eqn. (1) are potential term for SSB.
- 4th term in Eqn (1) is the kinetic energy term of the gauge field A_μ.
- Notice that there is no mass term for the gauge field A_μ as it would manifestly break the gauge invariance.

- The situation $\mu^2 > 0$: we have a vacuum at (0, 0). The exact symmetry of the Lagrangian is preserved in the vacuum: we have QED with a massless photon and two massive scalar particles Φ_1 and Φ_2 each with a mass μ.

- However, the situation becomes remarkably different, when $\mu^2 < 0$. Here, we have an infinite number of vacua, each satisfying, $\Phi_1^2 + \Phi_2^2 = -\frac{\mu^2}{\lambda} = \mathrm{v}^2$. The particle spectrum is obtained by studying the Lagrangian under small perturbations using the same procedure as for the continuous global symmetry . Because of local gauge invariance some important differences appear. Extra terms will appear in the kinetic part of the Lagrangian through the covariant derivatives. Using again the shifted fields η and ξ we define the vacuum as $\Phi_0 = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$.

- Kinetic Term : $(\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}^\mu\Phi)$

$$= \left((\partial_\mu - igA_\mu)\frac{1}{\sqrt{2}}(v+\eta+i\xi)\right)^\dagger\left((\partial_\mu - igA_\mu)\frac{1}{\sqrt{2}}(v+\eta+i\xi)\right)$$

$$= \left((\partial_\mu + igA_\mu)\frac{1}{\sqrt{2}}(v+\eta-i\xi)\right)\left((\partial_\mu - igA_\mu)\frac{1}{\sqrt{2}}(v+\eta+i\xi)\right)$$

$$= \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) + \frac{1}{2}g^2v^2A_\mu A^\mu + \frac{1}{2}g^2(\eta^2+\xi^2)A_\mu A^\mu - gv(\partial_\mu\xi)\mathrm{A}^\mu + \mathrm{int.\,terms}$$

- Potential Term: $\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$

$$= \frac{\mu^2}{2}(v+\eta-i\xi)(v+\eta+i\xi) + \frac{\lambda}{4}((v+\eta-i\xi)(v+\eta+i\xi))^2$$

$$= \frac{\mu^2}{2}\left((v+\eta)^2+\xi^2\right) + \frac{\lambda}{4}\left((v+\eta)^2+\xi^2\right)^2$$

$$= -\frac{\lambda v^2}{2}(v^2+\eta^2+\xi^2+2v\eta) + \frac{\lambda}{4}(v^2+\eta^2+\xi^2+2v\eta)^2$$

$$= -\frac{1}{2}[\lambda v^4 + \lambda v^2\eta^2 + \lambda v^2\xi^2 + 2v^3\eta] + \frac{\lambda}{4}[v^4+\eta^4+\xi^4+4v^2\eta^2+2v^2\eta^2+2v^2\xi^2+4v^3\eta+$$
$$2\eta^2\xi^2+4v\eta^3+4v\eta\xi^2]$$

$$= \lambda v^2\eta^2 + 0\cdot\xi^2 + int.terms$$

- Note that Kinetic term for Gauge field remains the same.

Hence, the transformed Lagrangian after SSB looks like :

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2 + \frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) + 0\cdot\xi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2v^2A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$- gv(\partial_\mu\xi)\mathrm{A}^\mu + Int.terms \quad \text{------------------------------------}(2)$$

- **Observations:**

Initially we have two real scalar fields Φ_1 and Φ_2 and a massless gauge field A_μ with two DOF (Since mass less only transverse modes are present). Total DOF is 4. After SSB we see that η field becomes massive with proper mass sign but ξ field becomes massless which is known as Goldstone boson already discussed in our last lecture. But due to local gauge invariance via SSB we are now able to generate the mass of the gauge field which is forbidden due to gauge invariance. The mass of the corresponding gauge field A_μ is gv .

Problem:

Although we have generated various interaction terms not explicitly written in Eqn.(2) The interaction term: $gv(\partial_\mu\xi)\mathrm{A}^\mu$ is of no physical meaning because it tells us that a real scalar field transforms into a gauge field without any other field.

- In a local gauge invariant theory we see that A_μ is fixed up to a term $\partial_\mu\theta$ as can be seen from previous expression. In general, A_μ and Φ change simultaneously. We can exploit this freedom, to redefine A_μ and remove all terms involving the ξ field.

- Considering terms involving $\partial_\mu\xi$ and A_μ we get:

$$\frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) - gv(\partial_\mu\xi)\mathrm{A}^\mu + \frac{1}{2}g^2v^2A_\mu A^\mu = \frac{1}{2}g^2v^2\left(A_\mu - \frac{1}{gv}(\partial_\mu\xi)\right)^2 = \frac{1}{2}g^2v^2{A'_\mu}^2$$

$$X^2 = X_\mu X^\mu$$

This specific choice, i.e. taking $\theta = -\frac{\xi}{v}$, is called the unitary gauge. Of course, when choosing this gauge (phase of rotation θ) the field Φ changes accordingly.

$$\Phi' = e^{-\frac{i\xi}{v}}\Phi = e^{-\frac{i\xi}{v}}\frac{1}{\sqrt{2}}(v+\eta+i\xi) \approx e^{-\frac{i\xi}{v}}\frac{1}{\sqrt{2}}(v+\eta)e^{+\frac{i\xi}{v}} = \frac{1}{\sqrt{2}}(v+h)$$

Keeping in mind that upto 1st order in ξ is taken into consideration

- Here we have introduced the real h-field. When writing down the full Lagrangian in this specific gauge, we will see that all terms involving the ξ-field will disappear and that the additional degree of freedom will appear as the mass term for the gauge boson associated to the broken symmetry.

Let us write down the Lagrangian in this unitary gauge :

$$\mathcal{L} = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda\left(\Phi^\dagger \Phi\right)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$= \left((\partial_\mu + igA_\mu) \frac{1}{\sqrt{2}} (v+h) \right) \left((\partial_\mu - igA_\mu) \frac{1}{\sqrt{2}} (v+h) \right) - \mu^2 \frac{1}{2} (v+h)^2 - \frac{\lambda}{4} (v+h)^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$= \frac{1}{2} \left(\partial_\mu h\right)(\partial^\mu h) - \lambda v^2 h^2 + \frac{1}{2} g^2 v^2 A_\mu A^\mu + g^2 v\, A_\mu A^\mu h + \frac{1}{2} g^2 v^2 h^2 A_\mu A^\mu - \lambda v\, h^3 - \frac{1}{4} \lambda h^4 + \frac{1}{4} \lambda v^4$$

Observations:

1. 1st two terms give the Higgs mass Lagrangian with correct mass signature.
2. 3rd term gives mass of the gauge field introduced due to local gauge invariance of the Lagrangian.
3. 4th and 5th terms give interaction among Higgs and gauge fields.
4. 6th and 7th terms give higgs self interactions
5. Last term is of no physical meaning.

Summary on breaking of a local gauge invariant theory:

We added a complex scalar field (2 degrees of freedom) to our existing theory and broke the original symmetry by using a ’strange’ potential that yielded a large number of vacua. The additional degrees of freedom appear in the theory as a mass term for the gauge boson connected to the broken symmetry (m_γ) and a massive scalar particle (m_h).

Take home Message:

A massless vector field carries two DOF (transverse polarizations); when A_μ acquires mass, it picks up a 3rd DOF (longitudinal polarization). Where did this extra DOF come from?

Answer: It came from the Goldstone boson, which meanwhile disappeared from the theory. The gauge field ‘ate’ the Goldstone boson, thereby acquiring both a mass and a third polarization state. This is the famous Higgs mechanism which is based on union of local gauge invariance and SSB.

In the Standard Model this mechanism is employed to generate masses of the weak bosons responsible for Weak Interactions.

Higgs Mechanism for Non-Abelian Gauge Theory

Higgs Mechanism for Non-Abelian Gauge Theory

Here we shall consider $SU(2)$ local Non-Abelian gauge transformation defined by $U(1) = e^{iT^a\theta^a(x)}$; where $T^a s$ are the generators of $SU(2)$ gauge group and $\theta^a s$ are the parameters of transformation, which are function of $(t, \vec{x})$.

Following the same procedures as adopted in case of Abelian case we can write the $\mathcal{L}$, the Lagrangian density for complex scalar field, which remains invariant under $SU(2)$ local gauge group as :

$$\mathcal{L} = (\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}^\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2 - \frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu} \quad \text{----------------------------} \quad (1)$$

Where, $\mathcal{D}_\mu = \partial_\mu + igT^aW^a_\mu$

Under $SU(2)$ gauge transformation the gauge field and the complex scalar field transforms in the following way:

$$W^a_\mu \rightarrow W^a_\mu - \frac{1}{g}\left(\partial_\mu\theta^a\right) - f^{abc}\theta^bW^c_\mu$$

$$\Phi \rightarrow \mathrm{e}^{iT^a\theta^a}\,\Phi$$

The Gauge field strength tensor is : $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - f^{abc}W^b_\mu W^c_\nu$

Higgs Mechanism for Non-Abelian Gauge Theory

Here, $\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$ with $\Phi^+ = \frac{1}{\sqrt{2}}\,(\Phi_1 + i\Phi_2)$ and $\Phi^0 = \frac{1}{\sqrt{2}}\,(\Phi_3 + i\Phi_4)$

$$\Phi^\dagger \Phi = \frac{1}{2}(\Phi_1^2 + \Phi_2^2 + \Phi_3^2 + \Phi_4^2)$$

Under SSB with $\mu^2 < 0$, one can choose the VEV to be : $\Phi_0 = \sqrt{\frac{1}{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}$ with proper choice of gauge. Here the minimum is chosen along the direction of the field Φ_3

In constructing the Standard Model we need Higgs Mechanism for Abelian as well as Non-abelian case.

Higgs Mechanism in the SM

The SM Gauge group is $SU(2)_L \times U(1)_Y$

In the SM the gauge group $SU(2)_L \times U(1)_Y$ breaks spontaneously to $U(1)_{em}$ gauge group. Here Y stands for hypercharge related by $Q = I_3 + \frac{Y}{2}$, with $Y = B + S$ where B and S denote Baryon number and Strangeness number respectively. The ingredients for Higgs Mechanism in this case are:

1. An isospin doublet , $\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\phi_4 \end{pmatrix}$

We can consider only $SU(2)_L \times U(1)_Y$ multiplets here ,i.e., Lagrangian would contain the terms which preserve these symmetries.

Here Left handed fermions (electron-neutrino) are put into doublets with Isospin , $I = \frac{1}{2}$ and right ones are considered to be singlets with $I = 0$. The upper component of the isospin doublet has $I_3 = \frac{1}{2}$ and for lower ones , $I_3 = -\frac{1}{2}$

Higgs Mechanism in the SM

- The Higgs potential is : $V(\Phi, \Phi^\dagger) = \mu^2 \Phi^\dagger \Phi + \lambda\left(\Phi^\dagger \Phi\right)^2$ with $\mu^2 < 0$.
- The covariant derivative for the SM Gauge group $SU(2)_L \times U(1)_Y$ is :

$$\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2} Y B_\mu + i\frac{g}{2}\tau^a W_\mu^a$$

- g' and g are gauge couplings for $U(1)_Y$ and $SU(2)_L$ gauge groups respectively.
- B_μ and W_μ^a are the gauge fields corresponding to $U(1)_Y$ and $SU(2)_L$ gauge groups respectively. Here a can run from 1-3. So we have one gauge field for $U(1)_Y$ gauge group and three gauge fields for $SU(2)_L$ gauge group.
- Y and τ^a are generators for the groups mentioned above. τ^as are Pauli matrices.
- The field strength tensors corresponding to the gauge fields are :

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - f^{abc} W_\mu^b W_\nu^c$$

Higgs Mechanism in the SM

- The choice of vacuum that breaks the symmetry of the Lagrangian Spontaneously generates masses of the gauge bosons following the lines of Higgs Mechanism. The vacuum picks up the minimum in one particular direction , i.e., $\Phi_3 = v$ and $\Phi_1 = \Phi_2 = \Phi_4 = 0$. The VEV is given to the real part of the neutral Higgs boson. Charged Higgs field does not acquire VEV because that would spoil the charge conservation.
- The Higgs field is defined after SSB as : $\Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ where h denotes the Higgs field.
- This vacuum is always neutral since $I = \frac{1}{2}$ and $I_3 = -\frac{1}{2}$ and we choose $Y = +1$,

  so that $Q = I_3 + \frac{Y}{2} = 0$.
- It can be shown that this choice of vacuum breaks the $SU(2)_L \times U(1)_Y$ gauge symmetry but leaves $U(1)_{em}$ symmetry invariant leaving **only** the photon field massless.
- The choice of vacuum is dictated by the choice of unitary gauge discussed in Abelian case.

Broken & Unbroken symmetries in the SM

- How do we check whether a symmetry associated with a gauge group is broken or intact after the SSB?

  The breaking of a gauge symmetry is ensured by the fact that if the vacuum state of the system after SSB does not respect the symmetry which the corresponding Lagrangian of the system respects. Invariance implies that $e^{i\alpha Z}\Phi_0 = \Phi_0$ with α being parameter of transformation and Z being the generator of the corresponding transformation. If we consider infinitesimal transformation, we obtain, $(1 + i\alpha \mathrm{Z})\Phi_0 = \Phi_0 \Rightarrow Z\Phi_0 = 0.$
- Let us check the condition for the gauge groups : $SU(2)_L$; $U(1)_Y$; $U(1)_{em}$
- $SU(2)_L$:

$$\tau_1\Phi_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = +\frac{1}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix} \neq 0 \rightarrow \text{Broken!}$$

$$\tau_2\Phi_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = -\frac{i}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix} \neq 0 \rightarrow \text{Broken!}$$

$$\tau_3\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \neq 0 \rightarrow \text{Broken!}$$

- $U(1)_Y$:

$$Y_1\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} = +\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} \neq 0 \rightarrow \text{Broken!}$$

Broken & Unbroken symmetries in the SM

- The breaking of two gauge symmetries implies that four gauge bosons (W_1, W_2, W_3 and B) acquire a mass through the Higgs Mechanism. However, it will be later shown that W_1 and W_2 will mix to form the charged gauge bosons, $W^{\pm}$ and W_3 and B will mix to form the neutral gauge bosons, Z and γ .
- Photon γ being massless, it implies that one remnant symmetry is there which is responsible for photon to be massless. Let us find that symmetry: The vacuum must be invariant under this remnant symmetry.
- The corresponding symmetry generator can be constructed from the generators τ_3 and Y.
- The generator is : $Q\Phi_0 = \frac{1}{2}(\tau_3 + Y)\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} = 0$ →Unbroken!!
- This result is not surprising because $U(1)_{em}$ is conserved as the vacuum is neutral and we have

$\Phi_0' = e^{i\alpha Q_{\Phi_0}} \Phi_0 = \Phi_0$