MODULE-6

- Parton Model
- Callan-Gross relation & Bjorken Scaling

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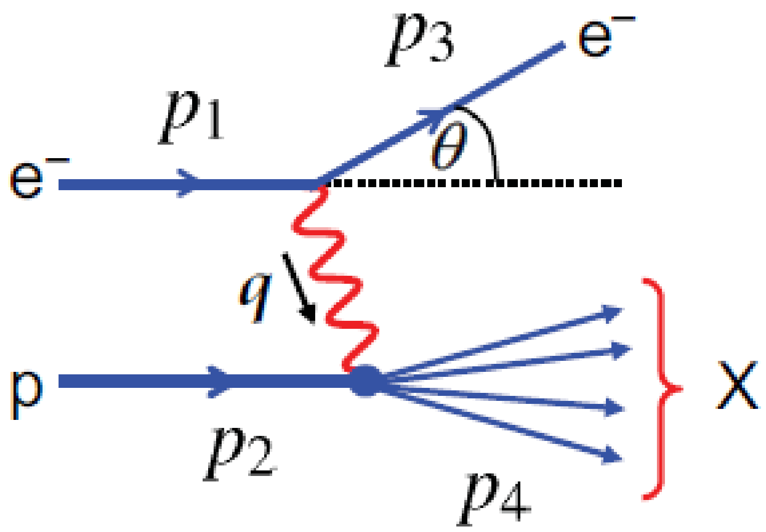
The Parton Model

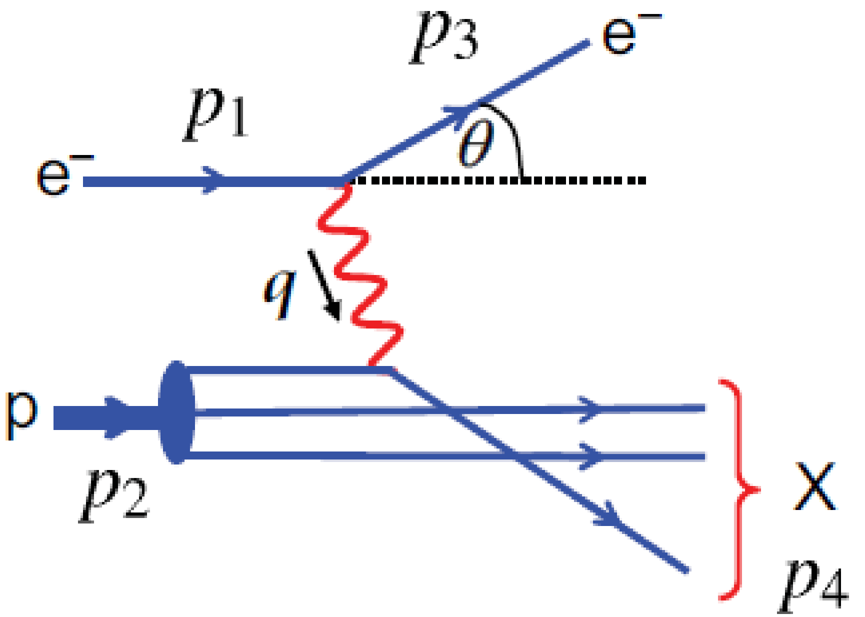
Continued…

Before quarks and gluons were accepted Feynman proposed that protons were made of point-like constituents called Partons.

Bjorken scaling and Callan-Gross relationship can be explained by assuming that DIS is dominated by the scattering of a virtual photon from point-like spin ½ constituents of the proton.



Scattering from a proton with structure functions



Scattering from a point-like quark within proton

- How do the above two pictures of interaction relate to each other?

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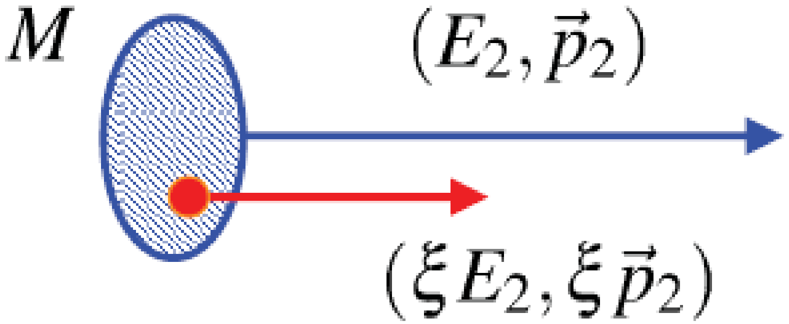
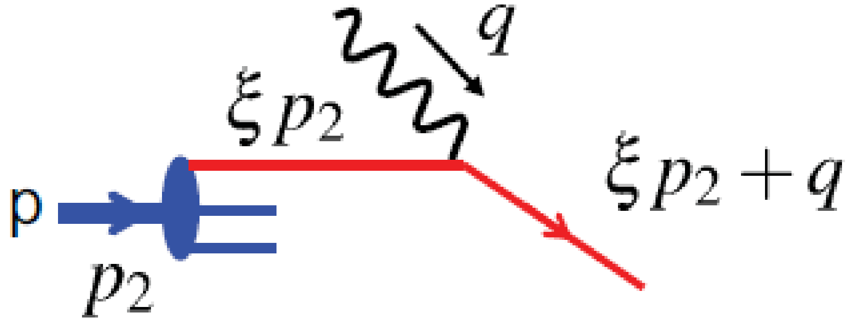
- In Parton model quarks are assumed to be quasi-free or free. In this model basic interaction is ELASTIC

  in which the electron scatters from point-like spin ½ free Dirac particles (quarks) residing within the proton.
- This model is formulated in a frame where proton has very high energy, often referred to as *Infinite momentum frame*, where we can neglect the mass of the proton and $p_2 = (E_2, 0,0, E_2)$
- In this frame masses of electron and quarks are negligible and any momentum transverse to the direction of the proton is neglected.
- Let the quark carry a fraction ξ of the proton's 4-momentum.





- After the interaction the struck quark's four momentum is.
- $(\xi p_2 + q)^2 = m_q^2 \approx 0 \quad => \quad \xi^2 p_2^2 + q^2 + 2\xi p_2 \cdot q = 0 \quad => \quad q^2 + 2\xi p_2 \cdot q = 0 => \xi = -\frac{q^2}{2\xi p_2 \cdot q} = x$
- $x = \frac{Q^2}{2\xi p_2 \cdot q}$; Bjorken x can be identified as the fraction of the proton momentum carried by the struck quark.

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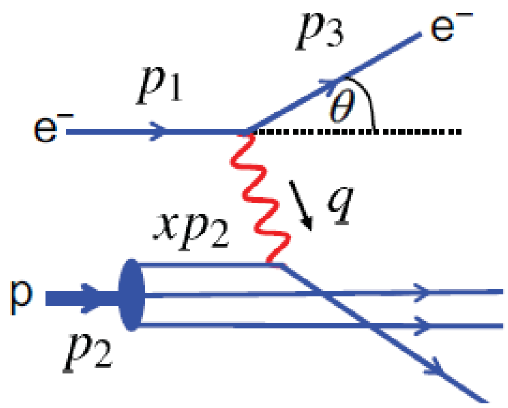
- In terms of proton momentum $s = (p_1 + p_2)^2 \approx 2p_1 \cdot p_2$; $y = \frac{p_2 \cdot q}{p_2 \cdot p_1}$
- For the underlying quark interaction
  $s_q = (p_1 + xp_2)^2 \approx 2xp_1 \cdot p_2 = xs$
  $y_q = \frac{xp_2 \cdot q}{xp_2 \cdot p_1} = y$
  $x = 1$ (Since elastic scattering is assumed as quarks do not break)



- Consider the cross-section for $e - \mu$ elastic scattering
  Apply the result for $e\ q\ \rightarrow e\ q$ scattering (charge of quark in units of e and $m_q^2 = x^2 p_2^2$)

$$\left(\frac{d\sigma}{d\Omega}\right)_{q,x} = \left(\frac{\alpha^2 e_i^2}{4E^2 \sin^4 \frac{\theta}{2}}\right) \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2m_q^2} \sin^2 \frac{\theta}{2}\right]$$

In ultra-relativistic limit, the LI cross-section becomes

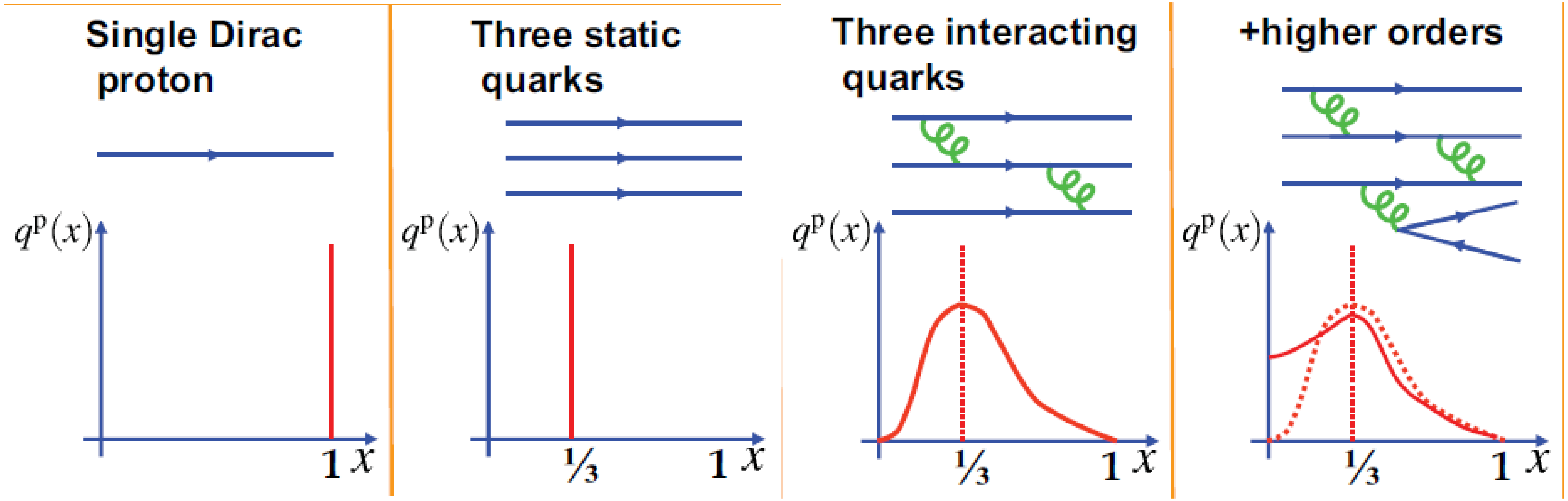
$$\left(\frac{d\sigma}{dQ^2}\right)_{q,x} = \frac{4\pi\alpha^2 e_i^2}{Q^4} \left[(1 - y) - \frac{y^2}{2}\right] ------- -(30)$$

Continued...

Eq. (30) represents the differential cross-section for elastic $e\ q$ scattering from a struck quark carrying a fraction x of the proton's momentum.

Now we need to account for the distribution of quark momentum within proton.

Introduce parton/quark distribution functions such that $q^p(x)\mathrm{dx}$ is the number of quarks of type q within a proton with momenta between x and $x + dx$.



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The cross-section for scattering from a particular quark flavor within the proton carrying momentum fraction x and $x + dx$ is

$$\left(\frac{d\sigma}{dQ^2}\right)_{q,x} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) - \frac{y^2}{2}\right] e_q^2 q^p(x)dx$$

Summing over all types of quarks within the proton gives the expression for the electron-proton scattering cross-section

$$\left(\frac{d^2\sigma}{dx\,dQ^2}\right)_{ep} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) - \frac{y^2}{2}\right]\sum_q e_q^2 q^p(x) \text{ ------- } -(31)$$

Compare the above equation with electron-proton scattering in terms of structure functions in ultra-relativistic regime (from Eq. (21))

$$\left(\frac{d^2\sigma}{dxdQ^2}\right)_{ep} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right] \text{ ------ } -(32)$$

Comparing Eq. (31) & (32) we obtain

$$F_2^{ep}(x,Q^2) = 2xF_1^{ep}(x,Q^2) = x\sum_q e_q^2 q^p(x) \text{ ------ } -(33)$$

Continued…

Eq. (33) shows the relation between measured structure functions and the underlying quark distribution functions .

The Parton Model predicts :

- Bjorken scaling: $F_2(x, Q^2) \rightarrow F_2(x)$ and $F_1(x, Q^2) \rightarrow F_1(x)$
- Callan-Gross Relation: $F_2(x) = 2xF_1(x)$
- Due to scattering from a spin ½ Dirac particles where the magnetic moment is directly related to the charge of the particle; hence electro—magnetic and pure magnetic terms are fixed with respect to each other.
- Note that present parton distributions can not be calculated from QCD because perturbation theory fails due to large strong coupling constant.
- Measurements of structure functions from cross-sections enable us to determine the parton distribution functions.
- Due to higher order contributions, the proton contains not only up and down type quarks (Valence quarks) but also anti-up and anti-down quarks (sea quarks) can also contribute. Heavier quark's contributions may be neglected.

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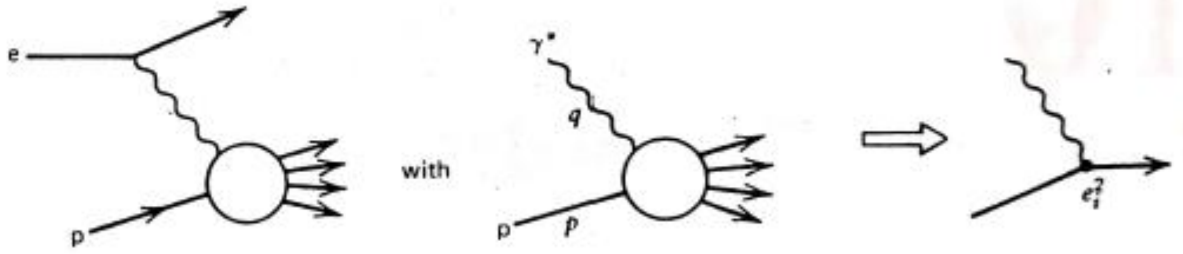

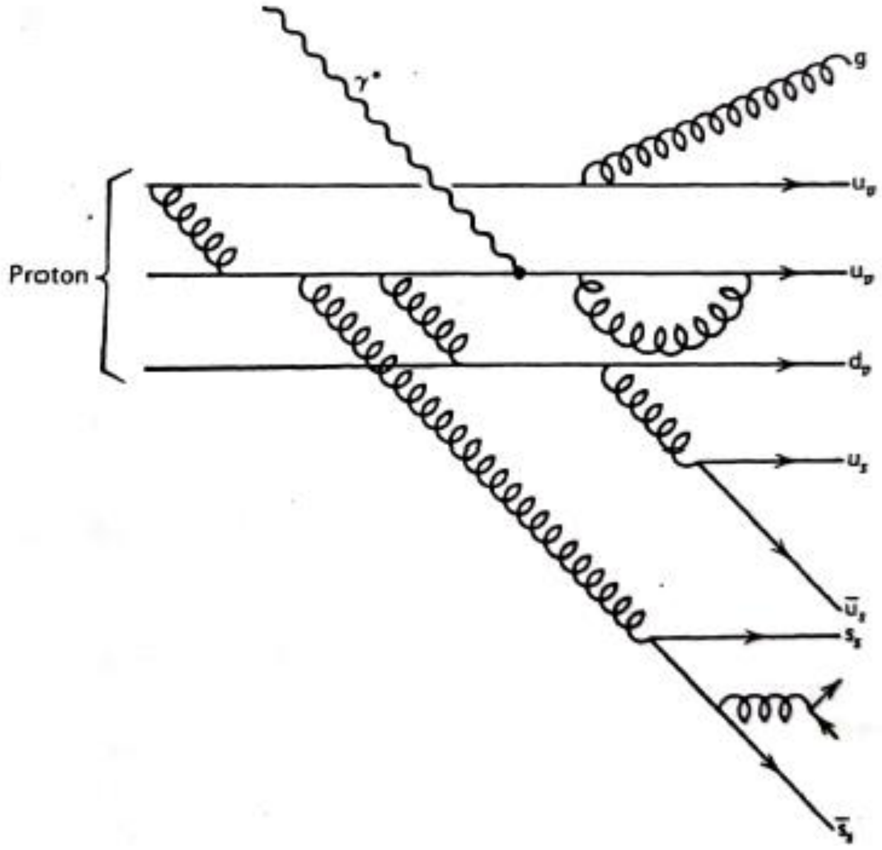

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For electron-proton scattering we have:

$$F_2^{ep}(x) = x\sum_q e_q^2 q^p(x) = x\left(\frac{4}{9}u^p(x) + \frac{1}{9}d^p(x) + \frac{4}{9}\bar{u}^p(x) + \frac{1}{9}\bar{d}^p(x)\right)$$

For electron-neutron scattering we have:

$$F_2^{en}(x) = x\sum_q e_q^2 q^n(x) = x\left(\frac{4}{9}u^n(x) + \frac{1}{9}d^n(x) + \frac{4}{9}\bar{u}^n(x) + \frac{1}{9}\bar{d}^n(x)\right)$$

Assuming isospin symmetry, $d^n(x) = u^p(x)$; $u^n(x) = d^p(x)$

Define the neutron distribution functions in terms of those of the proton

$$u(x) \equiv u^p(x) = d^n(x)\,;\, d(x) \equiv d^p(x) = u^n(x)\,;\, \bar{u}(x) \equiv \bar{u}^p(x) = \bar{d}^n(x)\,;\,;\, \bar{d}(x) \equiv \bar{d}^p(x) = \bar{u}^n(x)$$

$$F_2^{ep}(x) = 2xF_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right) ----(34)$$

$$F_2^{en}(x) = 2xF_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)\right) ----(35)$$

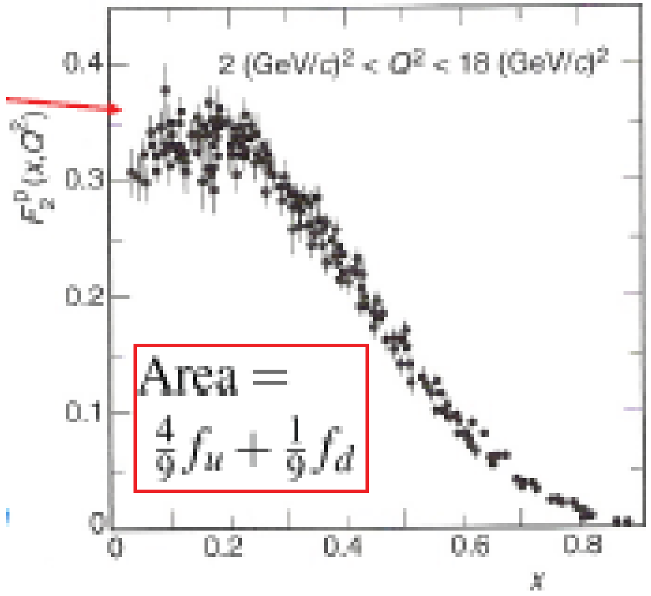
Continued…

Integrating Eq. (34) & (35) :

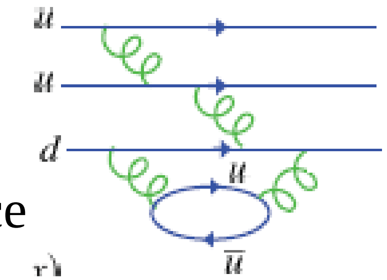
$$\int_0^1 F_2^{ep}(x)dx = \int_0^1 x\left(\frac{4}{9}[u(x)+\bar{u}(x)] + \frac{1}{9}\left[d(x)+\bar{d}(x)\right]d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right)dx = \frac{4}{9}f_u + \frac{1}{9}f_d$$

$$\int_0^1 F_2^{en}(x)dx = \int_0^1 x\left(\frac{4}{9}\left[d(x)+\bar{d}(x)\right] + \frac{1}{9}[u(x)+\bar{u}(x)]d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right)dx = \frac{4}{9}f_d + \frac{1}{9}f_u$$

- ❖ $f_u = \int_0^1 [xu(x) + x\bar{u}(x)]\,dx$ is the fraction of the proton momentum carried by the up and anti-up quarks.
- ❖ Experimentally:
- ❖ $\int_0^1 F_2^{ep}(x)dx \approx 0.18$ & $\int_0^1 F_2^{en}(x)dx \approx 0.12$
- ❖ Thus we get, $f_u \approx 0.36$ & $f_d \approx 0.18$
- ❖ In the proton, as expected, the up quarks carry twice the momentum of the down quarks.
- ❖ The quarks carry just over 50% of the total proton momentum. The rest is carried by gluons which being neutral doesn't contribute to electron-nucleon scattering.



Valence & Sea Quarks:



- Proton has complex structure difficult to explore completely
- The parton distribution function $u^p(x) = u(x)$ includes contributions from the valence quarks and the virtual quarks produced by gluons called the sea quarks.
- Splitting into valence and sea contributions :

$$u(x) = u_V(x) + u_S(x) \quad ; d(x) = d_V(x) + d_S(x) \quad ; \ \bar{u}(x) = \bar{u}_S(x) \ ; \quad \bar{d}(x) = \bar{d}_S(x)$$

- The proton contains two up quarks and one down quark and hence, $\int_0^1 u_V(x)dx = 2$; $\int_0^1 d_V(x)dx = 1$
- Note that for sea quarks there is no such expectation!
- But sea quarks arise from gluon splitting into quark/anti-quark pairs with $m_u = m_d$. It is reasonable to expect : $u_S(x) = d_S(x) = \bar{u}_S(x) = \bar{d}_S(x) = S(x)$ [Let]
- Hence Eq. (34) and (35) become

$$F_2^{ep}(x) = x\left(\frac{4}{9}u_V(x) + \frac{1}{9}d_V(x) + \frac{10}{9}S(x)\right) \quad \& \quad F_2^{en}(x) = x\left(\frac{4}{9}d_V(x) + \frac{1}{9}u_V(x) + \frac{10}{9}S(x)\right).$$

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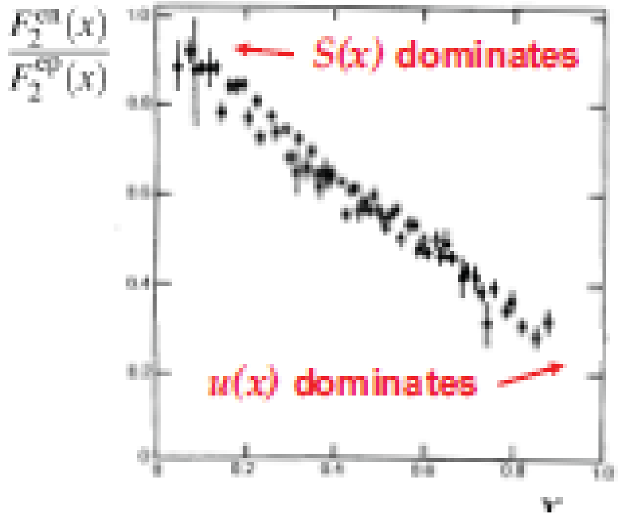
$$\frac{F_2^{ep}(x)}{F_2^{en}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

- The sea component arises from the process such as $g \to \bar{u}u$. Due to $\frac{1}{q^2}$ dependence of the gluon propagator, much more likely to produce low energy gluons. Hence the sea quarks comprise of low energy $\bar{q}/q$.
- Therefore, at low x, sea quarks are expected to dominate:

$$\frac{F_2^{ep}(x)}{F_2^{en}(x)} \to 1 \quad \text{as } x \to 0$$



Experimentally observed

- At high x, sea contributions are expected to be small:

$$\frac{F_2^{ep}(x)}{F_2^{en}(x)} \to \frac{4d_V(x)+u_V(x)}{4u_V(x)+d_V(x)} \quad \text{as } x \to 1$$

Using $u_V = 2d_V$ would give the ratio 2/3 as $x \to 1$

Experimentally : $\frac{F_2^{ep}(x)}{F_2^{en}(x)} \to \frac{1}{4}$ as $x \to 1$

Could not be understood!

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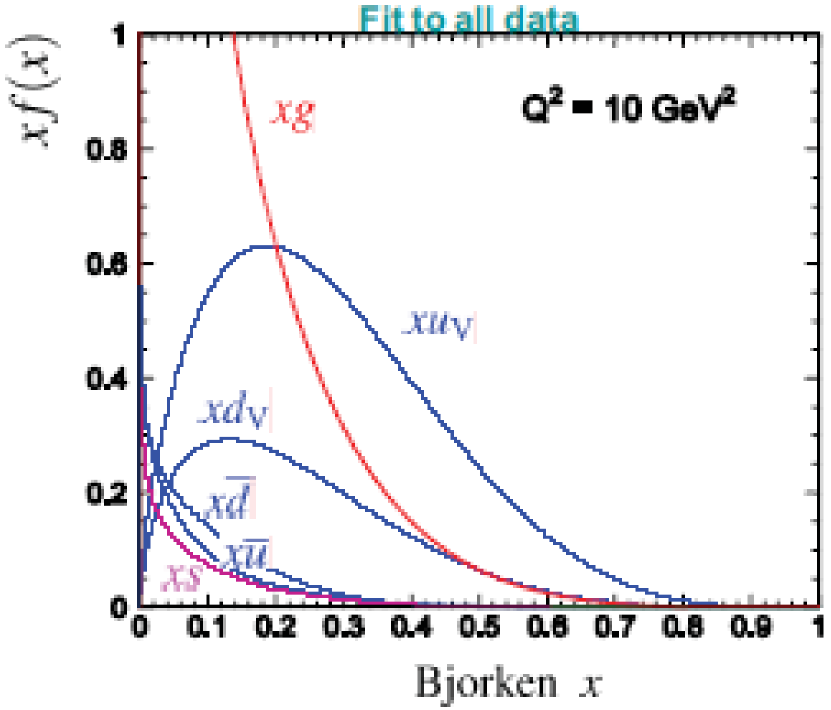
- Ultimately the parton distribution functions (pdf) are obtained from a fit to all experimental data including the neutrino scattering.
- Hadron-hadron collisions five information about gluon pdf denoted by $g(x)$



Note:

- Apart from at large x $u_V(x) \approx 2d_V(x)$
- For $x < 0.2$ gluons dominate
- In fits to data assume $u_s(x) = \bar{u}(x)$
- $\bar{d}(x) > \bar{u}(x)$ not understood – exclusion principle?
- Small strange quark component $s(x)$

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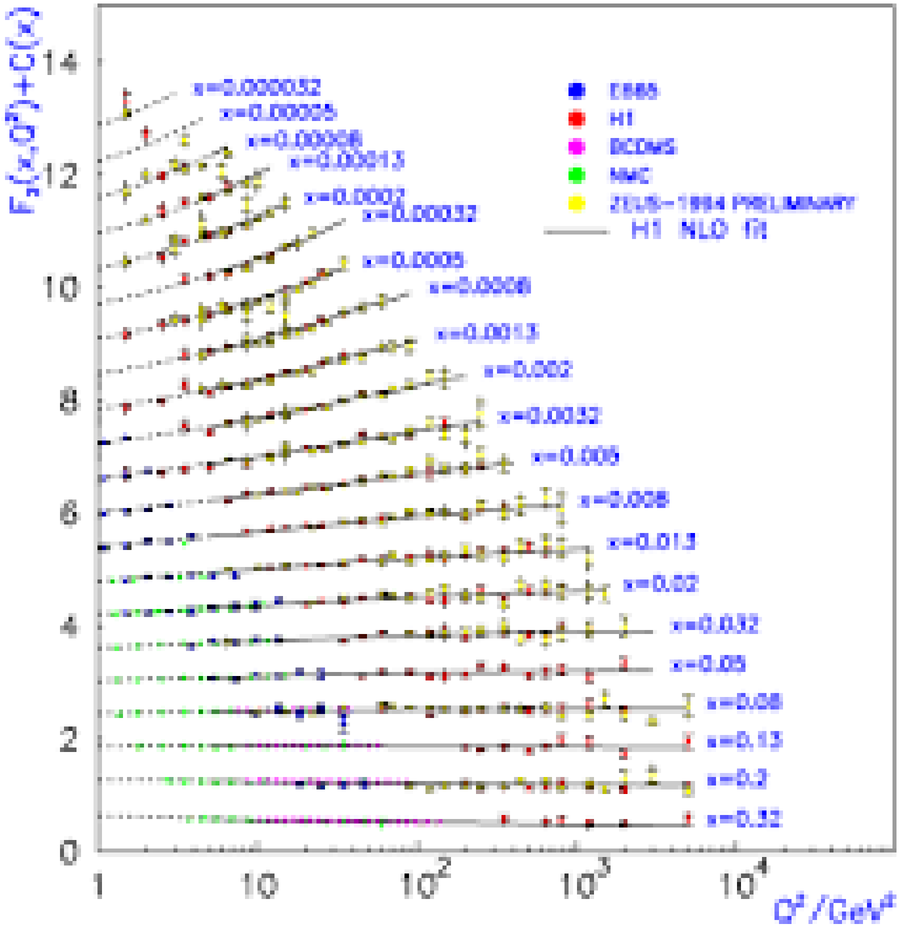
Scaling Violations: It is observed from the SLAC data (adjacent Figure) that the structure functions of the protons depend on Q^2 slowly for smaller values of x in the DIS regime. Thus Bjorken scaling is violated.



Origin: The existence of gluons inside protons and their subsequent interactions with quarks and among themselves lead to logarithmic dependence of structure functions on Q^2. It can be estimated from QCD calculations.

- For the last 50 years experiments have probed protons with virtual photons of ever increasing energy.
- Non-point like nature of scattering becomes apparent when λ_γ is of the order of size of the scattering center. $\lambda_\gamma = \frac{h}{|\vec{q}|}$

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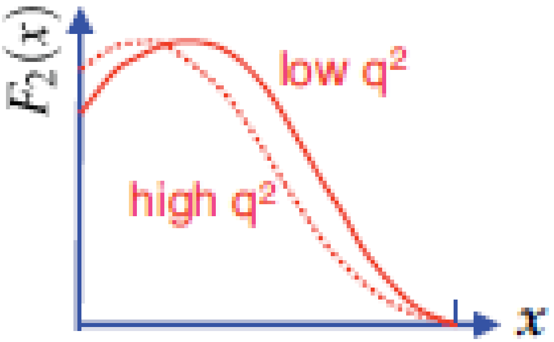
- Scattering from point-like quarks gives rise to Bjorken scaling. No q^2 dependence of cross-section.
- If quarks were not point-like, at high q^2, we would observe rapid decrease in cross-section with increasing q^2.
- For revealing quark substructure we must go to the highest q^2. No evidence has yet been observed. $R_{quark} < 10^{-18} m$.



- Observe small deviations from exact Bjorken scaling : $F_2(x) \rightarrow F_2(x, Q^2)$
- At high $Q^2 (= -q^2)$ observe more low quarks. Since at high Q^2(shorter wavelength) resolve finer substructure: i.e., reveal quark is sharing momentum with gluons. At higher Q^2 expect to see more low x quarks.
- QCD can not predict the x dependence of $F_2(x, Q^2)$.
- But can predict the Q^2 dependence of $F_2(x, Q^2)$.
- Measurements of structure functions not only provide powerful test of QCD the calculations of cross-sections at Hadron colliders (pp or $p\bar{p}$)

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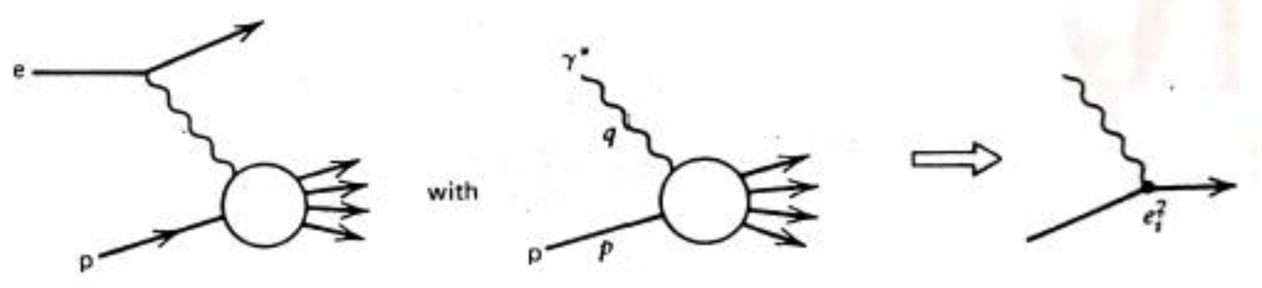
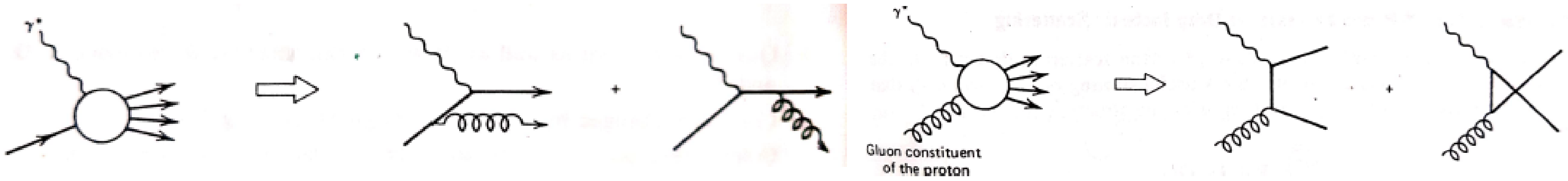
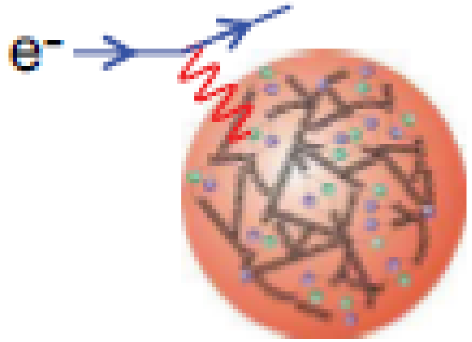
- The contributions from QCD processes, $(i)\ \gamma^* q \to qg\ \ (ii) \gamma^* g \to q\ \bar{q}$ to the process $ep \to eX$ have twofold Consequences: (1) Violation of scaling property, and (2) The direction of outgoing quark (hadron jet) will no longer be collinear with the virtual photon.
- Low x indicates that the momentum fraction carried by the struck quark is very small that is dominance of sea quarks in the structure functions.





Summary

- Elastic electron-proton scattering gives us information about the dimension of the proton via form factors.
- Form factors give us charge distribution and magnetic moment distributions of proton.
- At high energy inelastic scattering occurs and structure functions contain the information about constituents of proton.
- At very high electron energies $\lambda \ll r_p$, the proton appears to be a sea of quarks and gluons.



- DIS = Elastic scattering from quasi-free constituent quarks :
  - ❖ Bjorken Scaling $F_1(x, Q^2) \rightarrow F_1(x)$ => point-like scattering
  - ❖ Callan-Gross $F_2(x) = 2xF_1(x)$ => scattering from spin ½ particle
- Describe scattering in terms of parton distribution functions $u(x)$, $d(x)$ ….which describe momentum distribution inside a nucleon.
- The proton is a very complex system than just uud – a sea of quarks/gluons
- Quarks carry about 50% of the proton’s momentum – the rest is due to low energy gluons making the business messy.