MODULE-4

- Nuclear Reactions
- Compound Nucleus Hypothesis
- Energetics of Nuclear Reactions

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NUCLEAR REACTIONS(L1)

Nuclear Reactions

Excites states of a nucleus can be estimated by nuclear reactions.

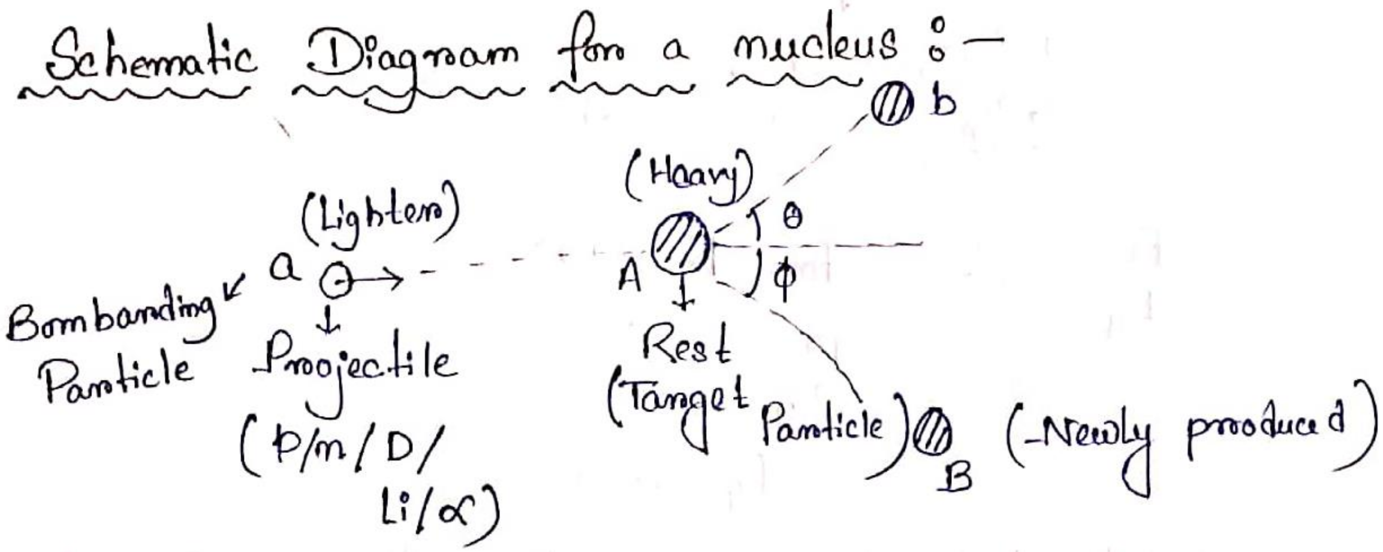
$A \rightarrow A^*$(Excited states of the nucleus A)

Nuclear Reactions are represented by $A(a, b)B$ where A , B are mother and daughter nuclei respectively and a, b are bombarding and the outgoing particle produced in the NR.

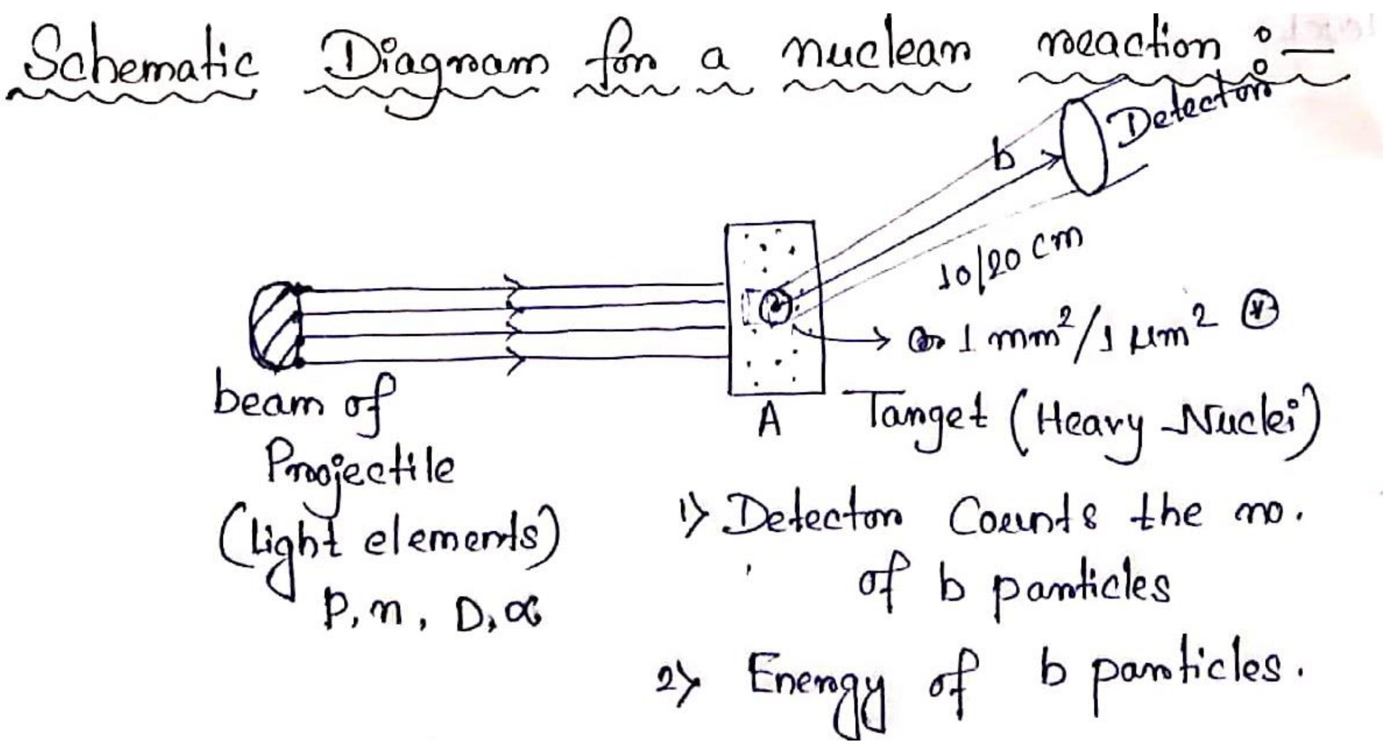
$A(a, b)B \Rightarrow A + a \rightarrow B + b$ $\quad$ $X(a, b)Y \Rightarrow X + a \rightarrow Y + b$

In all NR's mass number and atomic numbers are conserved.

Schematic Diagram of a Nuclear reaction:



- Target nucleus is clamped inside an evacuated chamber (low pressure).
- Interaction region can be considered as a point interaction w.r.t the distance between the point of interaction and the detector.
- The detector being placed at a particular angle, particles produced in all directions can not be detected.



What do you mean by a NR?

Cross-section measures the probability of occurrence of a particular NR.

Let, $dN =$ Number of particles detected in the detector in a fixed interval of time.

- $dN \propto I$ where, I= Intensity/flux of the incoming beam of particles which means number of bombarding particles falling on unit area of target in unit time
- $dN \propto N_t$ where, N_t is the number of target particles in the volume hit by the incident beam.
- $dN \propto d\Omega$ where, $d\Omega$ is the solid angle subtended at the point of interaction.

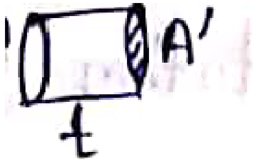
Continued…

$$dN \propto I\, N_t\, d\Omega$$

$$dN = \left(\frac{d\sigma}{d\Omega}\right) I N_t d\Omega$$

Where, $\frac{d\sigma}{d\Omega}$ is the differential scattering cross-section. It contains the information about the nature of nuclear interaction (strength of the interaction)

$$I\, N_t = I(A't)\, n$$



Where, n is the density of the target particles, A' is the area of cross-section and t is the thickness.

$$I\, N_t = \frac{N_i}{A'}(nt)A'$$

$$= N_i n_a$$

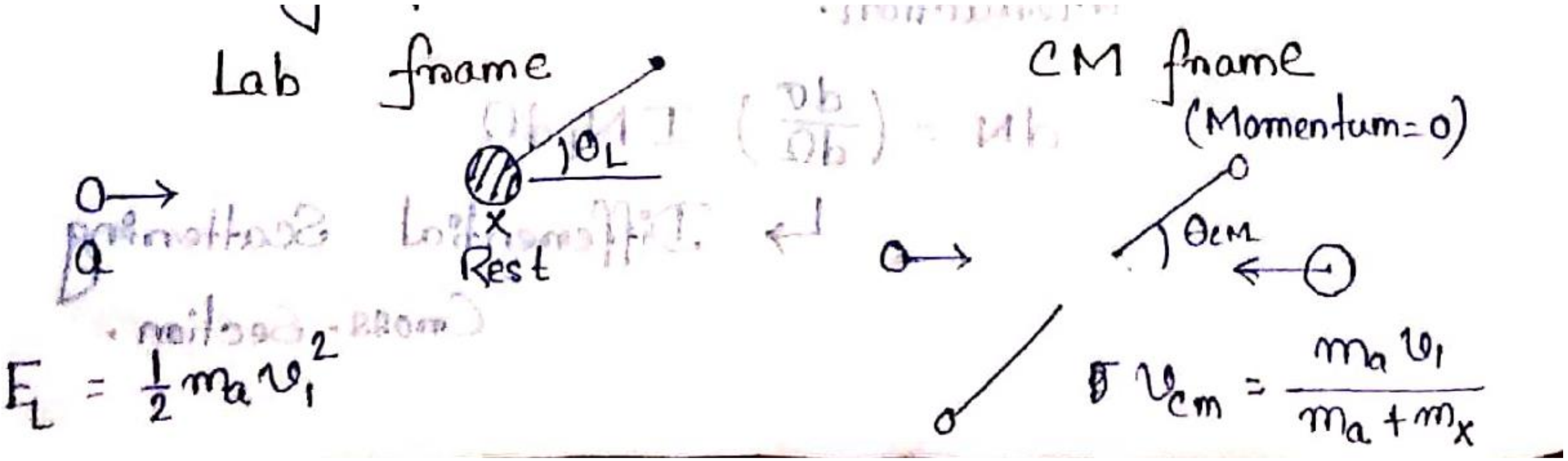
Where, n_a is the areal density of the target particles and N_i is the number of incident particles per unit time.

Now, $dN = \left(\frac{d\sigma}{d\Omega}\right) I N_t d\Omega$ [in terms of target particles]

$= \left(\frac{d\sigma}{d\Omega}\right) N_i n_a d\Omega$ [in terms of incident particles]

Dimension of $\frac{d\sigma}{d\Omega}$: $\left[\frac{d\sigma}{d\Omega}\right] = L^2$

- Nuclear cross-sections are very small, new unit is introduced. 1 $barn = 10^{-28} m^2$
- Only the target nuclei in the zone of interaction can interact with the bombarding particles.



$$E_{CM} = \frac{1}{2} m_a \left(\frac{m_X v_1}{m_X + m_a}\right)^2 + \frac{1}{2} m_X \left(\frac{m_a v_1}{m_X + m_a}\right)^2$$
$$= \frac{1}{2} m_a m_X (m_a + m_X) \frac{1}{(m_a + m_X)^2} v_1^2$$

$$= \frac{1}{2} m_a v_1^2 \frac{1}{(m_a + m_X)} = \frac{E_L m_X}{m_a + m_X} = \frac{E_L}{1 + \frac{m_a}{m_X}}$$

Types of Nuclear Reactions

1. Scattering : $a = b$; $X = Y$
2. Nuclear Reactions : $a \neq b$; $X \neq Y$

Scattering : $a = b$; $X = Y$

- Elastic Scattering $(k_a + k_X = k'_a + k'_X)$
- Inelastic Scattering

In Elastic scattering the daughter nucleus remains in its ground state

In Inelastic scattering the daughter nucleus goes to its excited state keeping its identity intact.

In Inelastic scattering the KE of the incoming particles is greater than that of the outgoing particles as aresult of which the daughter nucleus produced remains in excited state.

Depending upon the energy of the bombarding particles NR's can be classified into two types namely:

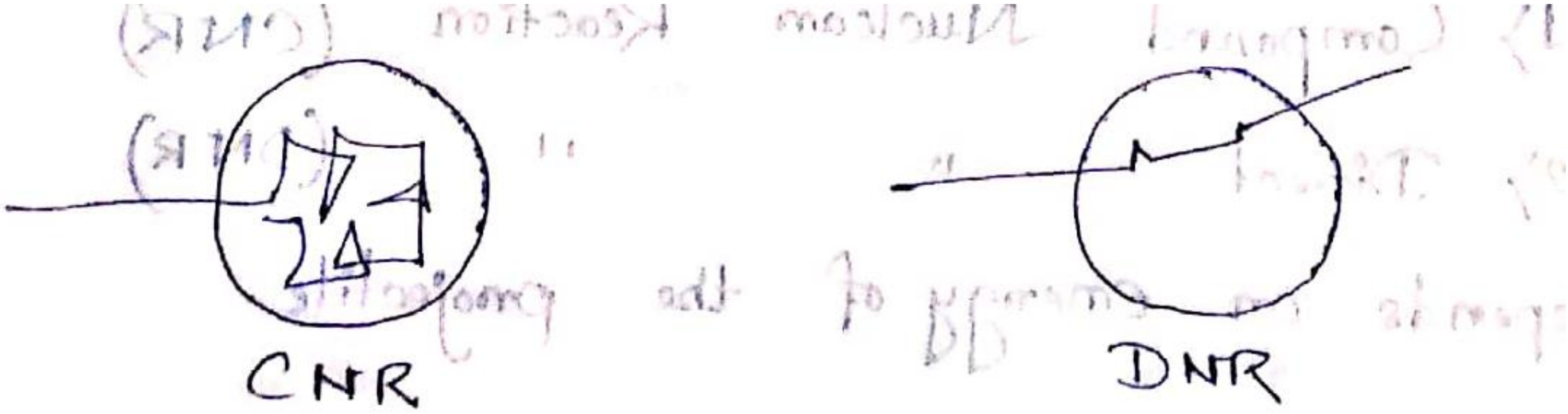
1) Compound Nuclear Reaction (CNR)
2) Direct Nuclear Reaction (DNR)

Depending upon the energy of the bombarding particles, it can interact with only a few nucleons on the surface of the nucleus or all the nucleons inside the nucleus and get sufficient time to be absorbed inside the nucleus to form a bigger nucleus.

It has been estimated that the average time for formation of a big nucleus is about 10^{-15} sec whereas the time required for interaction with few (two or three) nucleons with the projectile is about 10^{-22} sec.

In CNR the projectile $'a'$ transfers its energy to the nucleons by the process of collision between the projectile and the nucleons. If sufficient time is available for this phenomena to occur, the projectile gets trapped inside the nucleus and become a part of the nucleus. The newly formed nucleus is known as Compound Nucleus.

In generally the new nucleus is formed in its excited state : $a + X \rightarrow C^* \rightarrow b + Y$ [formation time is $10^{-15}\ s$]



How do we fix up the energy of the projectile at which DNR would occur or the energy at which CNR would occur?

$$\lambda = \frac{2\pi\hbar c}{\sqrt{2Km_pc^2}}$$

$$= \frac{2\pi \times 200}{\sqrt{2 \times 10 \times 1000}} \approx 8\ fm$$

Where, K denotes KE of the projectile in MeV, m_pc^2 is the mass of the proton in MeV

$\hbar c = 200$MeV-fm

For $K = 40$ MeV, $\lambda = 4\ fm$

If $K \geq 40$ MeV, chances of DNR would enhance but for $K = 10$ MeV, CNR would be more probable.

In terms of time scale of $10^{-15}s$ CNR would occur and the same of $10^{-22}s$, DNR would occur.

- Estimation of time scale for DNR:

$K = 20 \text{ MeV}$

$\frac{1}{2}mv^2 = 20$

$\frac{1}{2} \times 1000 \times v^2 = 20$

$v = 0.2c$

Therefore, $t = \frac{d}{v} = \frac{10\times10^{-15}}{0.2\times3\times10^{8}} \approx 10^{-22}s$

- For DNR the angular distributions of outgoing particles will be forward peaked and for CNR the corresponding distribution of outgoing particles will be equally probable in all directions thereby making the distribution Isotropic.

NUCLEAR REACTIONS(L2)

Compound Nuclear Reactions

Compound Nuclear Hypothesis:

According to the hypothesis when a compound nucleus is formed , it looses its past memory i.e., how the compound nucleus is formed. In other words, the decay products of the Compound nucleus do not remember its past history.

$$p + {}^{63}_{29}Cu \rightarrow {}^{64}_{30}Zn^{*}$$

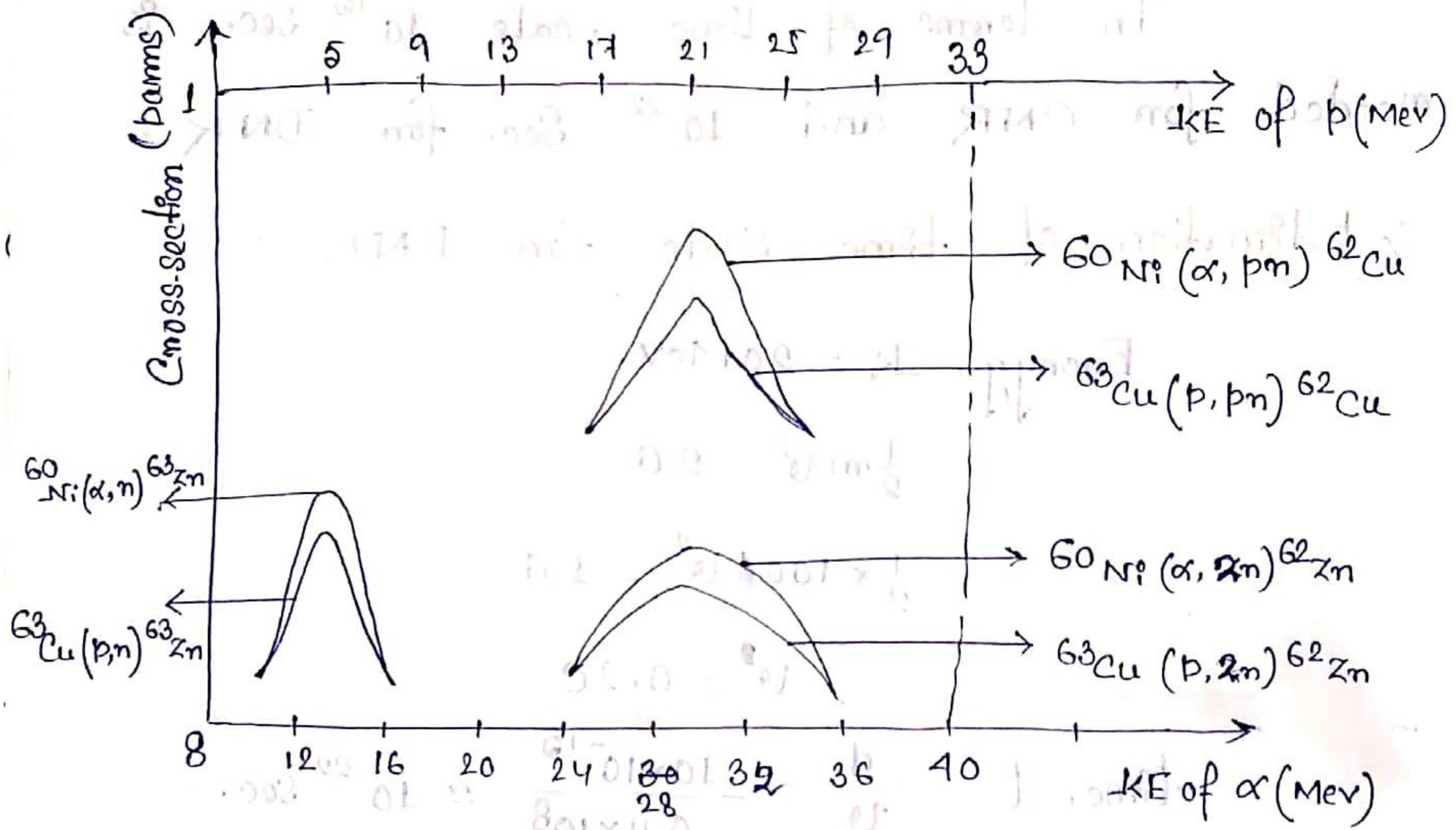
$$\alpha + {}^{60}_{28}Ni \rightarrow {}^{64}_{30}Zn^{*}$$

$${}^{64}_{30}Zn^{*} \rightarrow {}^{63}_{30}Zn + n$$

$$\rightarrow {}^{62}_{30}Zn + n + n$$

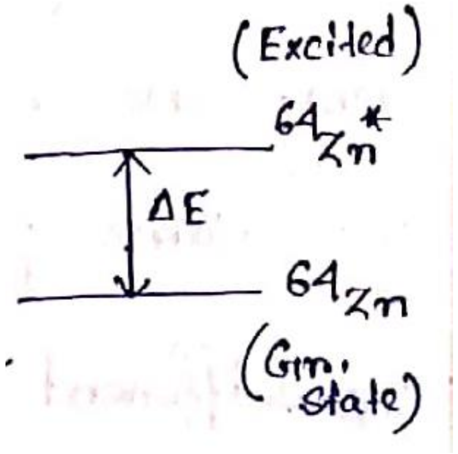
$$\rightarrow {}^{62}_{29}Cu + n + p$$

Compound Nuclear Reactions



Compound Nuclear Reactions

Conclusion: The distribution of decay products of the compound nucleus is independent of how the nucleus is formed.

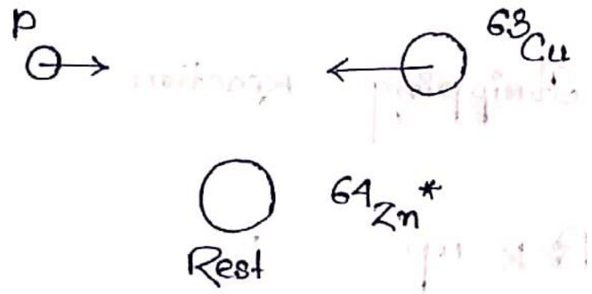


Consider some of the excited states of the nucleus.

Energy of the excited state = Ground state energy + ΔE (see the adjacent figure)

Excited energy happens to be the total energy of the nucleus.

Momentum Conservation



$$\vec{p} + \vec{p}_{Cu} = 0$$

$$\vec{p} = -\vec{p}_{Cu}$$

Energy Conservation

$$(m_p + m_{Cu})c^2 + K_p + K_{Cu} = m_{Zn}c^2 + \Delta E$$

$$(m_p + m_{Cu} - m_{Zn})c^2 + K_p + \frac{p^2 \times m_p}{2m_{Cu} \times m_p} = \Delta E$$

$$(m_p + m_{Cu} - m_{Zn})c^2 + K_p + \frac{K_p \times m_p}{m_{Cu}} = \Delta E$$

Compound Nuclear Reactions

$$(m_p + m_{Cu} - m_{Zn})c^2 + \left(1 + \frac{m_p}{m_{Cu}}\right) K_p \quad = \Delta E$$

When Cu is bombarded with protons we get,

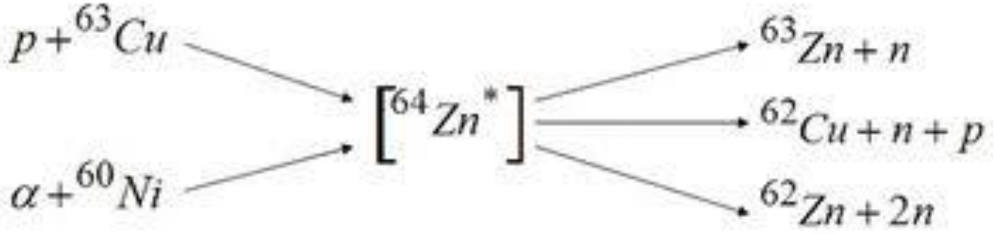
$$K_p = \frac{\Delta \mathrm{E} - (m_p + m_{Cu} - m_{Zn})c^2}{1 + \frac{m_p}{m_{Cu}}}$$

When Ni is bombarded with α-particles we get,

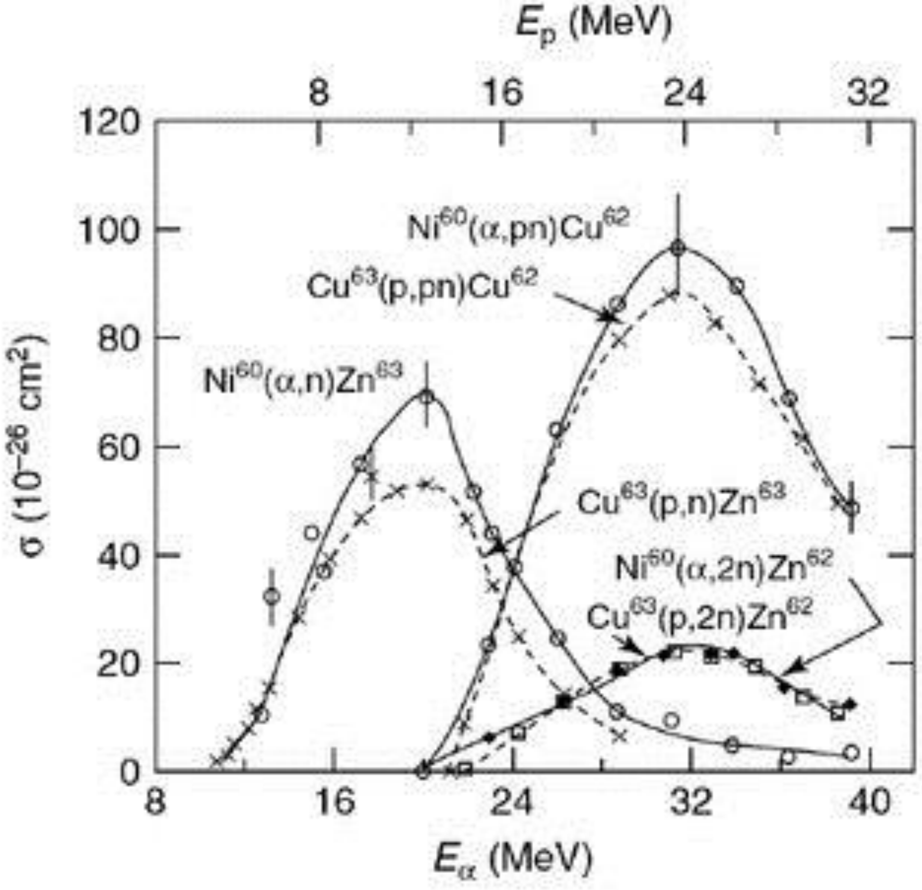
$$K_\alpha = \frac{\Delta \mathrm{E} - (m_\alpha + m_{Ni} - m_{Zn})c^2}{1 + \frac{m_\alpha}{m_{Ni}}}$$

Conclusion: In order to obtain the same excited state of a particular nucleus with projectiles, the KE of the projectiles would be different. Different choice of scales have been shown in the same plot.

Compound Nuclear Reactions



$$a + X \rightarrow [C^*] \rightarrow Y + b$$



Types of DNR

Stripping Reactions:

$X(d,p)Y$: $X + d \rightarrow Y + p$

$X(d,n)Y$: $X + d \rightarrow Y + n$

Pick up Reactions:

$X(p,d)Y$: $X + p \rightarrow Y + d$

Capture Reactions:

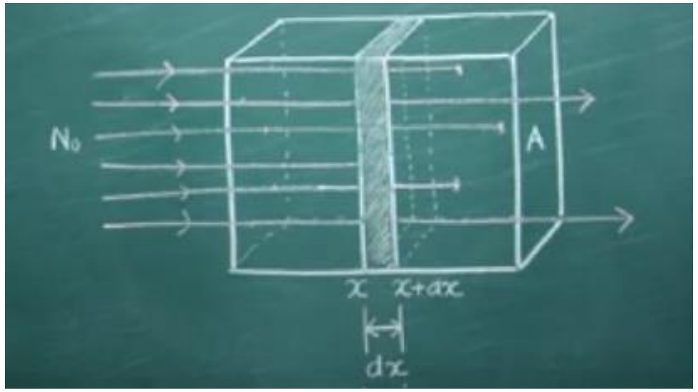
$X(n,\gamma)Y$: $X + n \rightarrow (Y^*) \rightarrow Y + \gamma$

Study the following Nuclear Reactions from S.N.Ghoshal's Book.

- α – induced NR
- p – induced NR
- $n -$ induced NR
- $\gamma -$ induced NR

Nuclear Cross-section

- Nuclear Reactions: Bombarding of particles on to target material in which nucleus being very small object, most of the bombarding particles passes through without any NR. A few can only interact with the nucleus leading to NR.
- How many of these incident particles interact with the target nuclei and lead to NR ?
- How many of the incident particles will penetrate through the material?
- Nuclear cross-section : It is the area around the nucleus on which if a projectile is incident, whether it would interact or not.
- Depends on nature of incident particles , energy of the incident particle.
- Greater the cross-section, more likely is the NR.



Consider a rectangular slab of target material. The area of the slab facing the incident beam is A. Consider a small slice of slab having thickness dx and cross-section A. N_0 is the number of incident particles entering one of the faces of the slab. How many of the NR's are actually taking place within this slab?

$N(x) =$ Number of incident particles surviving after crossing a distance x through the slab, i.e., the number of incident particles entering the left face of the slab of infinitesimal thickness dx.

$N(x) - dN =$ Number of incident particles leaving the right face of the slab of infinitesimal thickness dx.

$n =$ number density of target nuclei

Number of target nuclei within the infinitesimal slice of thickness $dx \quad = n\,A\,dx$

Area of cross-section $= n\,A\,dx\,\sigma$ [σ is the area corresponding to one nucleus]

$$\frac{No\ of\ interactions}{Total\ no\ of\ incident\ particles} = \frac{Total\ nuclear\ cross{-}section}{Area\ of\ cross{-}section\ of\ the\ slab}$$

$$\Rightarrow \frac{dN}{N} = \frac{n\sigma A\,dx}{A} = n\sigma\,dx$$

We assume that every single incident particle interacts only once.

$\int_{N_0}^{N} \frac{dN}{N} = -\int_0^x n\sigma\,dx$ [particle number decreases as they move along x]

$$ln\frac{N}{N_0} = -n\sigma x$$

$$N(x) = N_0 e^{-n\sigma x}$$

- The number of incident particles survived after moving through a distance x can be determined.
- It depends on σ, i.e., the cross-section, as the cross-section increases the decrease in number occurs exponentially.
- It depends on n, in a denser material the number of incident particles decreases more sharply than less denser material. Interaction probability enhances here.
- Rate of the NR can be determined.

Rate of NR for a thin slab of thickness Δx , $R = \frac{\Delta N}{\Delta t} = \frac{N_0 - N}{\Delta t} = \frac{N_0 - N_0 e^{-n\sigma\Delta x}}{\Delta t} = \frac{N_0}{\Delta t}\left(1 - e^{-n\sigma\Delta x}\right)$

$$R = \frac{N_0}{\Delta t}\left(1 - e^{-n\sigma\Delta x}\right)$$

For extremely thin slab, $\Delta \mathrm{x} \ll 1 \Rightarrow n\sigma\Delta x \ll 1$

In this limit, $R \approx \frac{N_0}{\Delta t}(1 - 1 + n\sigma\Delta x) = \frac{N_0 n\sigma\Delta x}{\Delta t}$

Flux of incident particles is denoted by ϕ which is the number of incident particles per unit area of cross-section per unit time.

$\phi = \frac{N_0}{A\Delta t}$

$\therefore R = \left(\frac{N_0}{A\Delta t}\right) \times A \times n\sigma\Delta x$

$\quad = \phi(A\, n\, \Delta x)\sigma$

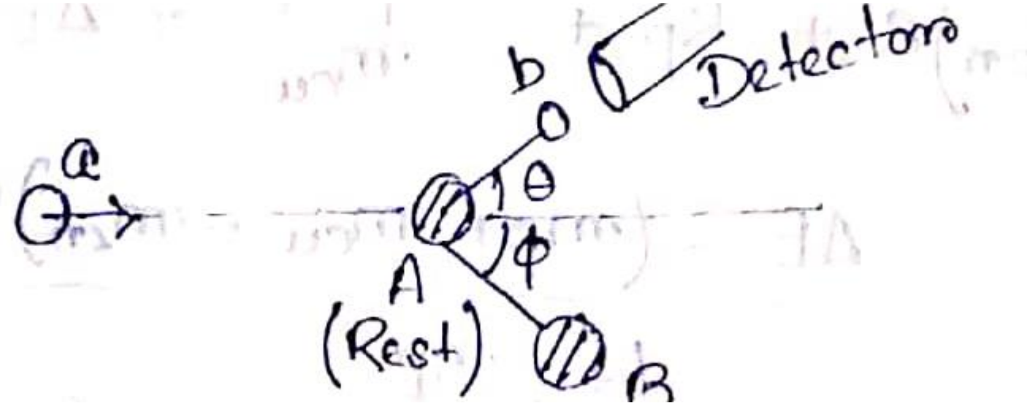
$\quad = \phi\, N'\sigma$ where, N' is the number of target particles within the thin foil.

Problem: The nuclear cross-section of ^{113}Cd for capturing thermal neutrons is 2×10^4 *barn* and the number of ^{113}Cd per cubic meter is 5.37×10^{27} $atoms\ m^3$. What fraction of incident beam of thermal neutrons is absorbed by a Cd sheet of 0.1 mm thickness. [Hint: Use $\frac{N_0 - N}{N_0}$] Ans. 68%

NUCLEAR REACTIONS(L3)

Energetics of Nuclear Reactions:

NR: $A + a \rightarrow B + b$



- A : Target nucleus (at rest)
- a : Projectile moving along +ve $x-$ direction with KE K_a
- B : Daughter nucleus recoiling with KE K_B at an angle ϕ with the direction of projectile a
- b : Outgoing particle is moving with KE K_b making an angle θ with +ve $x-$ direction. Detector is placed at an angle θ with +ve $x-$ direction.
- The daughter nucleus remained within the target, hence can not be estimated. K_B, ϕ should be eliminated from the equations.
- K_a is known parameter, masses of incoming (m_A, m_a) and outgoing (m_B, m_b) particles are known.
- K_b and θ could be estimated from the detector.

$$m_A, m_a, m_B, m_b, K_a, K_b, \theta$$

- K_

Assume the daughter nucleus to be in excited state with KE K_B : $A + a \rightarrow B^* + b$

Momentum Conservation:

$$\vec{p}_a = \vec{p}_B + \vec{p}_b$$

$$\vec{p}_B = \vec{p}_a - \vec{p}_b$$

$$p_B^2 = (\vec{p}_a - \vec{p}_b) \cdot (\vec{p}_a - \vec{p}_b)$$

$$p_B^2 = p_a^2 + p_b^2 - 2\vec{p}_a \cdot \vec{p}_b$$

$$p_B^2 = p_a^2 + p_b^2 - 2p_a p_b cos\theta \quad \text{--------} \quad (1)$$

Energy Conservation:

$$(m_A + m_a)c^2 + K_a = (m_B + m_b)c^2 + K_b + K_B + \Delta E \text{ --------} \ (2)$$

$Q -$ value of the NR:

$$Q = (m_A + m_a - m_B - m_b)c^2$$

Hence, we obtain,

$$Q + K_a = K_b + K_B + \Delta E$$

Assume the motion of daughter nucleus to be non-relativistic, i.e., $K_B = \frac{p_B^2}{2m_B}$

$$Q + K_a = K_b + \frac{p_B^2}{2m_B} + \Delta E \quad \text{----------} \ (3)$$

Continued….

Using Eq. (1) in Eq. (3), we get

$$Q + K_a = K_b + \frac{1}{2m_B}\left[p_a^2 + p_b^2 - 2p_a p_b cos\theta\right] + \Delta E$$

$$Q + K_a = K_b + \frac{m_a}{m_B}K_a + \frac{m_b}{m_B}K_b - \frac{2}{m_B}\sqrt{m_a m_b K_a K_b}cos\theta + \Delta E$$

$$\left(1 + \frac{m_b}{m_B}\right)\left(\sqrt{K_b}\right)^2 - \left(\frac{2}{m_B}\sqrt{m_a m_b K_a}cos\theta\right)\sqrt{K_b} - \left(\left(1 - \frac{m_a}{m_B}\right)K_a + Q - \Delta E\right) = 0 \text{ --------- } (4)$$

From the above equation one can estimate the excited states of the daughter nucleus.

Define : $A = \left(1 + \frac{m_b}{m_B}\right)K_b$

$$B = -\left(\frac{2}{m_B}\sqrt{m_a m_b K_a K_b}cos\theta\right)$$

$$C = -\left(1 - \frac{m_a}{m_B}\right)K_a - Q$$

Therefore, Eq. (4) becomes,

$$A + B + C + \Delta E = 0$$

Determining the values of A, B, C from experimental outcomes one can estimate different excited states from ΔE .

Problem: Consider the NR $^{16}O(d,p)^{17}O^*$ and estimate the 1st few excited states from the following given data. $K_a = 10\ MeV$, $Q = 1.96\ MeV$ and the detector is place at an angle $\theta = 25°$ with incoming direction of the projectile. Peaks are obtained at $K_b = 11.69\ MeV, 10.81\ MeV, 8.58\ MeV, 7.77\ MeV$. Assume mass numbers to be the masses of the nuclei. $m_a = 1, m_A = 16, m_b = 2, m_B = 17$.

For $K_b = 11.69\ MeV$

$$A = \left(1 + \frac{m_b}{m_B}\right) K_b = 12.38 \text{ MeV}$$

$$B = -\left(\frac{2}{m_B}\sqrt{m_a m_b K_a K_b}cos\theta\right) = -1.63\ MeV$$

$$C = -\left(1 - \frac{m_a}{m_B}\right) K_a - Q = -10.78\ MeV$$

Now , $A + B + C + \Delta E = 0 \Rightarrow \Delta E = 0.03\ MeV$ (Ground State energy)

For $K_b = 10.81\ MeV$

$A = 11.44\ MeV \quad B = -1.56\ MeV \quad C = -10.78\ MeV \quad \Rightarrow \Delta E = 0.91\ MeV$ (1st Excited State)

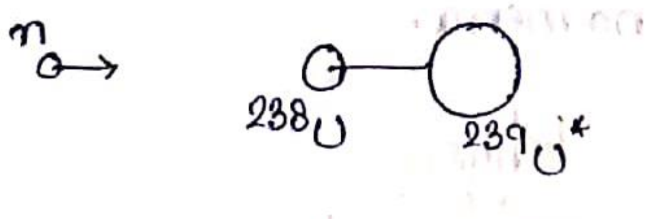
For $K_b = 8.58\ MeV$

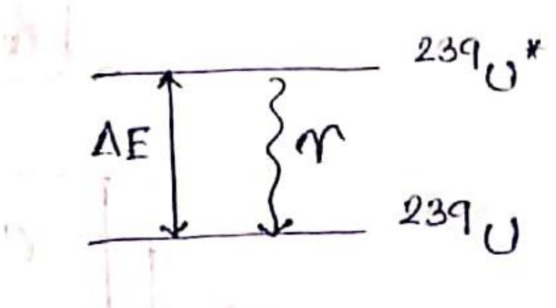
$A = 9.08\ MeV \quad B = -1.39\ MeV \quad C = -10.78\ MeV \quad \Rightarrow \Delta E = 3.09\ MeV$ (2nd Excited State)

Solve for 3rd excited state

Nuclear Resonance Reactions

$$n + {}^{238}U \rightarrow {}^{239}U^{*} \rightarrow {}^{239}U + \gamma$$





Momentum Conservation : $p_n = p_{^{239}U}$

Energy Conservation:

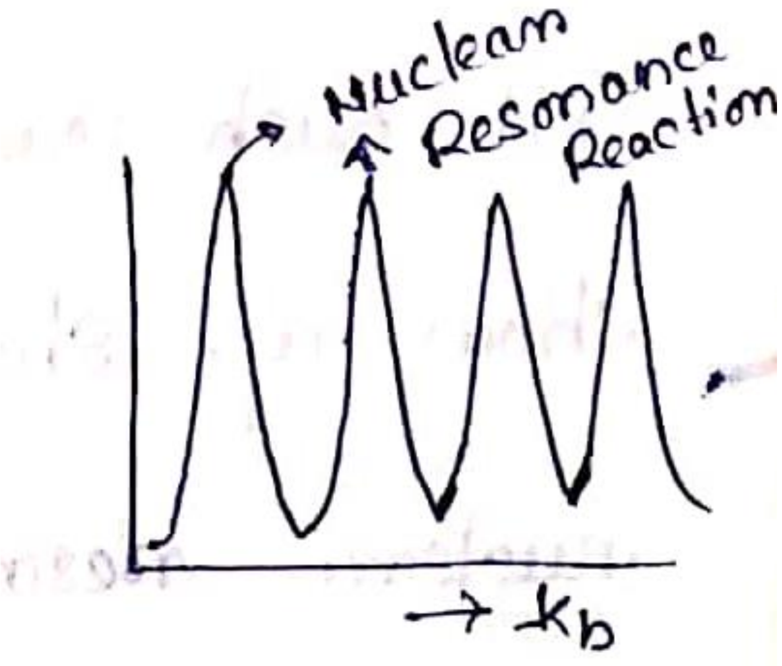
$$(m_n + m_{^{238}U})c^2 + K_n = m_{^{239}U}c^2 + \Delta E + \frac{p^2_{^{239}U}}{2m_{^{239}U}}$$

$$K_n + Q = \Delta E + K_n \frac{m_n}{m_{^{239}U}}$$

$$\Delta E = Q + K_n \left(1 + \frac{m_n}{m_{^{239}U}}\right)$$

For $K_n \geq 0 \Rightarrow \Delta E \geq Q$. Only excited states lying above Q value can be determined by this method.

- It is known that $Q = 4.86\ MeV$ for the given NR.
- Only excited states lying above $4.86\ MeV$ can be determined from this NR.
- Depending upon the values of K_n, one can determine the higher excited states of the daughter nucleus.
- Nuclear Resonance occurs when values of K_n are such that ΔE matches with certain excited states of energy levels of the daughter nucleus.
- At such particular values of K_n, the cross-section suddenly shows up sharp peak leading to the Nuclear Resonance phenomena.



Problem: In a NR represented by $X(a,b)Y$ with known $Q -$ value and X at rest,

a) Show that $\sqrt{K_b} = \dfrac{\sqrt{m_a m_b K_a}\cos\theta \pm \{m_a m_b K_a \cos^2\theta + (m_Y+m_b)[m_Y Q + (m_Y - m_a)K_a]\}^2}{m_Y + m_b}$

b) When $Q < 0$, there is a minimum value of K_a for which the reaction is not possible. Show that the threshold energy is given by $K_{th} = (-Q)\dfrac{m_Y+m_b}{m_Y+m_b-m_a}$.

c) When $Q < 0$, and $K_a > K_{th}$, K_b can take on two values. The maximum K_a that this can occur is $K_a{}'$.

Show that $K_a' = (-Q)\dfrac{m_Y}{m_Y - m_a}$

$\boldsymbol{Q} - value$ of a NR ($Q > 0 \Rightarrow Exoergic$ & $Q < 0 \Rightarrow Endoergic$)

$A + a \rightarrow B + b$

1. $Q = (m_A + m_a - m_B - m_b)c^2$
2. $Q = K_B + K_b - K_A - K_a$
3. $Q = B_B + B_b - B_A - B_a$

- Total energy of a relativistic particle = $mc^2 + K$
- $K_{th} = -Q\left[1 + \frac{m_a}{m_A}\right]$
- $K_{th} = -Q\left[\frac{m_1+m_2+m_3+m_4+\cdots}{2m_2}\right]$ $[1 + 2 \rightarrow 3 + 4 + 5 \ldots]$

  $E_{th} = K_{th} + m_1c^2 + m_2c^2$

Non-relativistic Scenario:

$X \rightarrow a + b$ [X at rest]

$$Q = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$Q = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\left(\frac{m_1v_1}{m_2}\right)^2 = \frac{1}{2}m_1v_1^2\left(1 + \frac{m_1}{m_2}\right) = K_a\left(1 + \frac{m_1}{m_2}\right)$$

$$K_a = \left(\frac{m_2}{m_1+m_2}\right)Q$$

Consider α-decay: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha + Q$

$$K_\alpha = \left(\frac{A-4}{A-4+4}\right)Q = \left(\frac{A-4}{A}\right)Q$$

$$K_Y = \left(\frac{4}{A-4+4}\right)Q = \left(\frac{4}{A}\right)Q$$

Problem: Calculate the threshold KE to produce π −meson from the reaction: $p + p \rightarrow p + p + \pi^0$. Given $m_p c^2 = 938\ MeV, m_\pi c^2 = 135\ MeV.$

$$Q = -135\ MeV$$

$$K_{th} = -Q\left[\frac{4m_p + m_\pi}{2m_p}\right] = 280\ MeV$$

Problem: If in a spontaneous α − decay of ${}^{232}_{92}U$ at rest, the total energy released in the reaction is Q. What would be the energy carried by the α −particle?

$${}^{232}_{92}U \rightarrow {}^{228}_{90}Th + {}^{4}_{2}\alpha$$

$$K_\alpha = \left(\frac{228}{228+4}\right)Q = \frac{57}{58}Q$$